ANALYSIS OF A MULTIPLEX-BUS SYSTEM

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Abstract: A bus is a collection of one or more wires. A bus is
called a multiplex-bus when it is shared by a number of
devices. There are many applications of a multiplex-bus. For
example, a microprocessor-based measurement system may
use a bus (say IEEE-488, VME, etc.) to interconnect a number
of microprocessor-based instruments, e.g., voltmeter,
ammeter, data acquisition devices, etc. All the automobile
companies are planning to use multiplex-buses (multiplex
wiring) in their future vehicles. Advanced electronic features
like antilock braking, electronic steering, and electronically
controlled transmissions and suspensions can improve
performance. But the wiring, components, and electronic
control units required for these systems must fight for space in
today's small cars, where they may invite diagnostic and repair
problems. Multiplexing can reduce wiring complexity and
eliminate redundant parts, while simplifying diagnostics. The
effectiveness of a multiplex-bus has been reported in a
number of papers [1-5].

As more and more devices are connected to a bus, the speed
of the bus reduces due to capacitive loading effect of the
devices. Hence, when too many devices are to be connected to a
bus, it is necessary to know the maximum rate (e.g., bits/sec)
at which data transfer can occur. Pulse width modulation
(PWM) technique is widely used to transmit digital data in
many industrial and consumer electronics applications. We
have developed an analytical model for a multiplex-bus system,
which will allow us to determine the shape of the pulses of
PWM signals at different nodes of the bus. We have also
developed a novel algorithm to convert our analytical model into
a software package. Currently we have developed software
packages for both IBM and Macintosh II personal computers.
The software accepts the following parameters as inputs:
electrical parameters (e.g., capacitance, resistance, etc.) of the
bus and the devices, bit pattern of the PWM signals, and the
characteristics of the bits (e.g., pulse period, pulse widths for
logic 0 and 1, etc.). The software displays the pulses at
different nodes of the bus. By looking at the display one can
easily determine whether or not the speed of transmission is
too high to detect the bits correctly by the receiving nodes. Our
algorithm is very novel and fast.

In this paper we will show our analytical model and the
novel algorithm in detail. Our model and algorithm will help
other people to develop software packages for the analysis of
multiplex-bus systems. This software package will help to
determine the maximum number of devices which could be
connected to a given bus if a certain minimum transmission
speed has to be maintained.

1. INTRODUCTION

Multiplexing has found wide application in many fields in
recent years through the power of modern electronics and
microprocessors [1-6]. Multiplexing technology has developed
primarily as the result of the needs of the communications
industry. Nearly all multiplexing developments are the result
of two major benefits: cost reduction and space saving. A bus is
called a multiplex-bus (multiplex wiring) when it is shared
by a number of devices. A microprocessor-based measurement
system may use a multiplex-bus to interconnect a number
of microprocessor-based instruments, e.g., voltmeter, ammeter,
data acquisition devices, etc. Multiplex-buses are going to be
used in the future automobiles. Advanced electronic features
like antilock braking, electronic steering, and electronically
controlled transmissions and suspensions can improve
performance. But the wiring, components, and electronic
control units required for these systems must fight for space in
today's small cars, where they may invite diagnostic and repair
problems. Multiplexing can reduce wiring complexity and
eliminate redundant parts, while simplifying diagnostics. The
effectiveness of multiplexing in automobiles are reported in a
number of papers [2,3,5,8]. The use of a multiplex-bus makes
a system very flexible. Implementing changes in car models and
the consequent function and load changes become very easy:
simply connect one or more additional processors (nodes) on
the same bus with proper software modification.

The problem with a multiplex-bus is that the bus speed
decreases as the system expands. Because as the system is
expanded by adding more devices (processors) the capacitive
loading effect on the bus increases. This increased capacitive
loading effect reduces the speed of the bus. Hence, when too
many devices are to be connected to a bus, it is necessary to
know the maximum rate (e.g., bits/sec) at which data transfer
can occur. Pulse width modulation (PWM) technique is widely
used to transmit digital data in many industrial and consumer
electronics applications. Time or frequency-domain analysis
can be used to evaluate the performance of a multiplex-bus
system. Time-domain analysis is very complex when
non-homogeneous devices (i.e., devices with different
electrical characteristics) are connected to a bus. But
time-domain analysis can be more attractive than
frequency-domain analysis when a bus is shared by
homogeneous devices. Estin described the data handling capacity
of a real transmission line (non-multiplex bus) [7]. In this
paper we have developed a novel technique to determine the
performance of a multiplex-bus system which is shared by
similar processors or processors with similar electrical
characteristics. We have also developed a novel algorithm to
translate our technique into a software package. Currently we
have developed software packages for both IBM and Macintosh II
personal computers. The software accepts the following
parameters as inputs: electrical parameters (e.g., capacitance,
resistance, etc.) of the bus and the devices, bit pattern of the
PWM signals, and the characteristics of the bits (e.g., pulse
period, pulse widths for logic 0 and 1, etc.). The software
displays the response of the pulses. By looking at the display
one can easily determine whether or not the speed of
transmission is too high to detect the bits correctly by the
receiving nodes. Our algorithm is very novel and simple. Our
time-domain analysis is verified by frequency-domain
analysis.
II. MODELING AND ANALYSIS OF A MULTIPLEX-BUS

An approximate electrical model of a bus and the devices can be shown by an electrical circuit with lumped parameters. If the devices are connected very close to each other (say less than a meter), a lumped parameter model of the bus would be as good as a distributed parameter model [9]. In order to make the analysis simple we assumed that the devices are uniformly distributed over the bus. A lumped parameter model of a multiplex-bus is shown in Figure 1.

![Figure 1: A lumped parameter model of a multiplex-bus system.](image)

Let us define a parameter \( Y \) as:

\[
Y = (LD + R) \cdot (CD + G)
\]

where \( D \) means \( \frac{d}{dt} \)

It can be shown that the relationship between the source voltage, \( V_0 \), and the voltage at node \( n \) can be expressed as [10]:

\[
V_n = f_n(Y) \cdot V_n
\]

where \( f_n(Y) \) is called an \textit{n-degree multiplex polynomial}, and it can be expressed as:

\[
f_n(Y) = \sum_{i=0}^{n} \binom{n+1}{i} Y^i
\]

where \( \binom{n+1}{i} = \frac{a!}{(a-b)! \cdot b!} \)

Notice that equation (2) is a differential equation, and we must solve this equation in order to write a time-domain expression for \( V_n \). Once a time-domain expression is known for \( V_n \), we can easily determine the maximum rate at which a source can supply the bits (pulse width modulated signals) such that certain predetermined specifications are satisfied. Two simple specifications may be as follows:

1. Threshold voltage to detect the presence of a pulse.
2. Minimum and maximum widths of a pulse for considering the pulse as a logic 1 or logic 0.

Equation (2) can be solved using either frequency-domain or time-domain techniques. Sometimes the time-domain analysis could be very complex, if not impossible, depending on the complexity of the equation. This paper presents the performance of multiplex bus systems. The performance was measured by solving equation (2) using a time-domain technique. A novel time-domain analysis of equation (2) exists due to following nice properties of the polynomial \( f_n(Y) \).

The roots of the polynomial \( f_n(Y) \), shown in Table 1, can be used for any multiplex-bus system which has a maximum of 16 nodes. For a system with more than 16 nodes, we need the root of \( f_n(Y) \) for \( n > 16 \). Since the same roots can be used for any multiplex-bus system, these roots can be kept in a file to be used by a program to determine the performance of different multiplex bus systems.

Let \( Y_i, 1 \leq i \leq n, \) be a root of the polynomial \( f_n(Y) \). Then we can write

\[
X_1^2 + 2t X_1 + q_1 = 0
\]

where \( X_1 = RCD, t = \frac{RG}{2} + \frac{NC}{2L} \), and

\[
q_1 = \frac{(RG-Y_i) \cdot X_i}{L}
\]

The roots of equation (4) can be written as:

\[
X_1(i) = -t + \left( t^2 - q_1 \right)^{0.5}
\]

When \( X_1(1) \) and \( X_1(2) \) are complex quantities we can define two terms: \( U_1 \) and \( \theta_1 \) as:

\[
U_1 = \left( t^2 + b_1^2 \right)^{0.5} \text{ and } \theta_1 = \tan^{-1} \left( \frac{b_1}{t} \right)
\]

The time domain expression for \( V_n \) can then be determined using the following two algorithms.

\section*{Algorithm 1:}

Make two matrices \( A \) and \( Z \) as follows:

\( \text{In this algorithm the notation } [A][j] \text{ is used to mean the element at ith row and jth column of matrix } A, \text{ and the notation } [Z][j] \text{ is used to mean the ith element of matrix } Z. \)

1. \( N = 0, i = 0. \)
2. \( i = i + 1; \)
3. \( \text{IF } X_1(1) \text{ and } X_1(2) \text{ are complex numbers THEN } N = N + 1. \)
4. \( \text{IF } i = n \text{ THEN go to Step #3. OTHERWISE Repeat Step #2.} \)
5. \( \text{Row} = 1, m = 0, L = N \)
6. \( \text{Row} = \text{Row} + 1, m = m + 1. \)
7. \( \text{IF } L > 0 \text{ THEN } L = L - 1 \text{ and } m = m + 1. \)
8. \( \text{Col} = 0, \text{i} = 1. \)
9. \( \text{IF } X_1(1) \text{ and } X_1(2) \text{ are complex numbers THEN } \text{Algorithm 2} \)
10. \( \text{THESE 2} \)

\section*{Algorithm 2:}

\( \text{Compute a matrix } K \text{ as } K = A^{-1} Z. \)

**Use algorithm 2 to determine a time-domain expression for } V_n.**
Algorithm-2: Procedure To Determine The Time-Domain Expression for $V_n$

1. Determine the values $X_l(1)$ and $X_l(2)$, for $1 \leq i \leq n$, using equation (15).
3. Determine the coefficients $k_l$, $1 \leq j \leq 2n-N$, using equation (21). Where $N$ is the number of complex pairs of $X_l(1)$ and $X_l(2)$, $1 \leq i \leq n$.
4. $i=0$, $j=0$.
5. $i=i+1$, $j=j+1$.
   IF $X_l(1)$ and $X_l(2)$ are complex numbers
   THEN $a_l = k_l$
   OTHERWISE $a_l = k_l$, $j=j+1$, $b_l = k_l$
   IF $i=n$ THEN go to Step #6,
   OTHERWISE Repeat Step #5.
6. Determine the time-domain expression for $V_n$ using the following equation.

$$V_n(t) = k_0 + \frac{e^{-xt}}{C} \sum_{l=1}^{N} a_l \cos(b_l t)$$

for complex $X_l(1)$ and $X_l(2)$

$$\sum_{l=1}^{N} \left( b_l(1) e^{X_l(1) t} + b_l(2) e^{X_l(2) t} \right)$$

for real $X_l(1)$ and $X_l(2)$

III. RESULTS FROM THE ANALYSIS

Figures 2-13 show the performances of multiplex bus systems for different values of the model parameters: $R$, $C$ and $G$. The performance was measured based on the following specifications.

1) Let $V_a$ be the amplitude of an undistored pulse. Then the threshold voltage ($V_{th}$) to detect the presence of a pulse is assumed to be greater than or equal to $0.5V_a$, i.e., $V_{th} \geq 0.5V_a$.

2) The pulse widths of an undistored one and an undistored zero are $T_1$ and $T_0$, respectively. The pulse period $T = 3T_1$.

Figure 2: Data rate versus capacitance (C) for a system with 4 nodes.

Figure 3: Data rate versus capacitance (C) for a system with 8 nodes.

Figure 4: Data rate versus capacitance (C) for a system with 16 nodes.

Figure 5: Data rate versus capacitance (C) for a system with 32 nodes.

$= 1.5T_0$. The threshold value of the width ($T_{1/2}$) for a one was taken as $0.5T_1 < T_{1/2} < T_1$. Similarly, the threshold value of the width ($T_{0/2}$) for a zero was taken as $0.6T_0 < T_{0/2} < T_0$. 

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Figure 2 shows the performance of a system with four nodes. This figure shows the data transmission rate (bits/sec) versus the model parameter $C$ (capacitance) for some fixed values of $G$ (conductance). Figures 3, 4 and 5 show similar results for systems having eight, sixteen and thirty two nodes, respectively. Figures 6, 7, 8 and 9 show the data transmission rate (bits/sec) versus the number of nodes in a system for different values of the model parameter $C$ and a value of $G = 6.667 \times 10^{-10}$ mho. Figures 10, 11, 12 and 13 show the similar results but for a value of $G = 3.333 \times 10^{-8}$ mho. The results presented in this paper will help one to determine the maximum number of devices which could be connected to a
given bus if a certain minimum transmission speed has to be maintained.

one can easily develop a software to determine the performance of a multiplex-bus. This software will help to determine the given specifications at a certain transmission rate. Similarly, devices such that the worst case response of a pulse satisfies some maximum number of devices that can be connected to a system evaluate the performance of a multiplex-bus system.

Figure 12: Data rate versus the number of nodes (32 to 48 nodes) in a system for 1/G = 3000 ohms.

Figure 13: Data rate versus the number of nodes (48 to 64 nodes) in a system for 1/G = 3000 ohms.

IV. CONCLUSIONS

This paper presents a novel time-domain analysis to evaluate the performance of a multiplex-bus system. A novel algorithm is also presented in the paper. Using this algorithm one can easily develop a software to determine the performance of a multiplex-bus. This software will help to determine the maximum number of devices that can be connected to a system such that the worst case response of a pulse satisfies some given specifications at a certain transmission rate. Similarly, the software can be used to determine the maximum transmission rate of a certain bus with certain number of devices.

REFERENCES


Table 1: Roots of the multiplex-polynomial \( f_n(x) \), for \( 1 \leq n \leq 16 \).

<table>
<thead>
<tr>
<th>n</th>
<th>Roots of the multiplex-polynomial ( f_n(x) )</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.0000</td>
</tr>
<tr>
<td>2</td>
<td>-2.6180, -0.3820</td>
</tr>
<tr>
<td>3</td>
<td>-2.2470, -1.5550, -0.1981</td>
</tr>
<tr>
<td>4</td>
<td>-3.5321, -2.3473, -1.0000, -0.126</td>
</tr>
<tr>
<td>5</td>
<td>-3.6825, -2.8368, -1.7154, -0.6930, -0.0630, -0.0581</td>
</tr>
<tr>
<td>6</td>
<td>-3.7709, -3.1361, -2.2411, -1.2908, -0.8030, -0.5302, -0.0981</td>
</tr>
<tr>
<td>7</td>
<td>-3.9271, -3.5881, -2.6180, -1.7905, -1.0000, -0.3820, -0.0437</td>
</tr>
<tr>
<td>8</td>
<td>-3.9649, -3.4762, -2.8915, -2.1945, -1.4527, -0.7967, -0.2941</td>
</tr>
<tr>
<td>9</td>
<td>-3.9916, -3.5763, -3.0393, -2.4910, -1.8548, -1.1966, -0.6456, -0.2411</td>
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<tr>
<td>10</td>
<td>-4.0111, -3.6252, -3.2410, -2.7307, -2.1495, -1.5500, -1.0000, -0.5339, -0.1791, -0.0223</td>
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<td>16</td>
<td>-3.5631, -3.9467, -3.8523, -3.4797, -3.1601, -2.8389, -2.4715, -2.0955, -1.7164, -1.3495, -1.0000, -0.6939, -0.4729, -0.2235, -0.0810, -0.0091</td>
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