Location of maximum shear (ACI 11.1.3.1)

Vr computed at a distance 'd'

(a) Beam.

(b) Shear force diagram.

Only valid if:
- Support reaction compresses the beam end
- Loads are applied at the top of beam
- No concentrated loads within 'd' distance from support face
**Critical Section Examples**

- (a) Beam loaded on tension flange.
- (b) Beam column joint.
- (c) Beam supported by shear.
- (d) Beam with concentrated load close to support.

**Example**

\[
D = 1.3 \text{ kips/ft} \\
L = 1.8 \text{ kips/ft}
\]

- (a) Elevation.
- (b) Section.

- \( f'_c = 4,000 \text{ psi} \)
- \( f_y = 60,000 \text{ psi} \)
- \( f_{y5} = 40,000 \text{ psi} \)
\[ w_u = 1.2 (1.3 \text{k/ft}) + 1.6 (1.6 \text{k/ft}) \]

\[ \text{ACI 9.2 w, T, H, Lr, S & R} = 0 \]

\[ w_u = 4.12 \text{k/ft} \]

\[ w_{D_u} = 1.56 \text{k/ft} \]

\[ \text{Load cases} \]

(c) Load case 1.

(d) Load case 2.

(e) Shear force envelope.
1) \( V_n \geq \frac{V_u}{\phi} \)

![Triangle diagram with labeled values](image)

\[ \frac{V_u}{\phi} \cdot d = 82.4 - \frac{a}{15} (82.4 - 12.8) \]
\[ = 73.1 \text{ kips} \]

2) Are stirrups required?

\[ V_c = 2\sqrt{f_{nc} \cdot d \cdot w} \]
\[ = 2\sqrt{4000 \cdot (12)(24)/1000} \]
\[ = 36.4 \text{ kips} \]

\[ V_n = 73.1 \text{ kips} > \frac{V_c}{2} = \frac{36.4}{2} = 18.2 \text{ kips} \]

Stirrups are required

3) Is cross-section ok?

\[ V_{s_{max}} = 8\sqrt{f_{nc} \cdot b \cdot d} = 4V_c \]
\[ V_{n_{max}} = V_c + V_{s_{max}} = 5V_c = 182 \text{ kips} \leq 73.1 \text{ kips} \]

Section OK!
4. Check stirrup anchorage & max. spacing

Assume 3 double leg stirrups $f_y = 40,000$ psi
$A_v = 0.22 \text{ in}^2$

A. Anchorage

ACI 12.13.2.1 allows 90° or 135° hook around a longitudinal bar
provide 2 x 4 along upper corners of stirrups for anchorage

B. Maximum spacing

0.5d or 24''

I half the distance if $V_s > 4V_c b d$ but
check if $V_n > V_c + 4V_c b_d = 109 K - no!$

Minimum spacing is 0.5d or 12''

Minimum $A_v$: ACI Eq. 11-13

$A_v \text{min} = 0.75 \sqrt{f_c b_d} \frac{b_w}{f_y}$

but not less than $A_v \text{min} = \frac{50 b_w}{f_y}$

or $S_{\text{max}} = \frac{A_v f_y}{0.75 \sqrt{f_c b_w}} = 15.5''$ for $A_v = 0.22$

and $S_{\text{max}} = \frac{A_v f_y}{50 b_w} = 14.7''$

$\therefore S_{\text{max}} = 12''$
5. Compute minimum stirrup spacing

\[
S = \frac{A_{stir} \phi}{V_u/V_c - V_c} \quad V_c = 36.4 \, k \quad V_u/V_c = 73.1 \, k
\]

\[
S = \frac{(0.22)(40)(24)}{(73.1 - 36.4)} = 6.1''
\]

Use 6" as distance from support face.

Increase spacing to 8" when sufficient and increase to S\text{max} = 12" when appropriate.

\[
\frac{V_u}{\phi} \text{ when } S = 8''
\]

\[
\frac{V_u}{\phi} = \frac{A_{stir} \phi}{S} + V_c
\]

\[
= \frac{(0.22)(40)(24)}{8} + 36.4 = 62.8 \, k
\]

\[
X = \frac{82.4 - 62.8}{82.4 - 12.8} \times 15' = 40.18' \text{ or } 50.1'' \text{ from beam end}
\]

\[
\frac{V_u}{\phi} \text{ when } S = 12''
\]

\[
\frac{V_u}{\phi} = \frac{(0.22)(40)(24)}{12} + 36.4 = 57 \, k
\]

\[
X = 72.6'' \text{ from beam end}
\]
Stirrups can be discontinued when

\[ \frac{Vu}{\phi} = \frac{Vc}{2} = 18.2'' \]

\[ x = \frac{82.4 - 18.2}{82.4 - 12.8} (15)(12) = 166'' \text{ from beam end} \]

No. 3, Grade-40 U stirrups with 135° hooks on upper ends.
Tapered beams

(a) Roof beam.

(b) Negative moment region of haunched beam.

(c) Haunched simply supported beam.

Examples

\[ V = V_r + C \tan \alpha_c + T \tan \alpha_T \]

\[ C = T = \frac{M}{Jd} \]

\[ V_r = V - \frac{M}{Jd} \tan \alpha \]

\[ \alpha = \alpha_c + \alpha_T \]
Deep beams

ACI 10.7.1

a) loaded on one face and supported on the opposite so that compression struts can develop between loads and supports

b) having either

i) clear span \( L \leq 4h \) (beam height)

ii) regions loaded w. concentrated loads within 2d from the face of the support.

Committee 445 suggested load should be high enough to cause 30% or more of the reaction at support

ACI 11.8.2 require deep beams to be designed by nonlinear analyses or by strut-and-tie models.

\[ \begin{align*}
\text{Diagram 1:} & \quad \begin{array}{c}
\downarrow d \\
\uparrow \quad p
\end{array} \\
\Delta & = 2d \text{ or less}
\end{align*} \]

\[ \begin{align*}
\text{Diagram 2:} & \quad \begin{array}{c}
\uparrow h \\
\downarrow \Delta \\
\downarrow p \leq 4h
\end{array}
\end{align*} \]