Shear in RC concrete beams

Flexural & shear stresses

Principal stresses

Principal compressive stress trajectory

Portion of the cracked RC beam
\[ T = \frac{M}{Jd} \quad T + \Delta T = \frac{M + dM}{Jd} \quad \Sigma \tau_x = 0 \]

\[ \Delta T = \frac{dM}{Jd} \quad \frac{dM}{dx} = V \]

\[ \Delta T = \frac{V}{Jd} \quad \frac{\Delta T}{dx} = \nu \]

\[ \frac{\Delta T}{dx} = \frac{V}{Jd} \quad \frac{\Delta T}{dx_b_w} = \nu \]

\[ \nu = \frac{V}{b_w Jd} \quad \text{if } Jd \rightarrow 0 \]

\[ \nu = \frac{V}{b_w d} \quad \text{as represented in ACI} \]
\[
V = \frac{d}{dx} (T Jd)
\]
down the beam since \( Jd \) is not constant

\[
V = \frac{d}{dx} T Jd + d \left( Jd \right) \frac{d}{dx} T
\]

when \( Jd \) is constant \( \frac{d (Jd)}{dx} = 0 \)

\[
V = \frac{dT}{dx} \cdot Jd
\]
\( \frac{dT}{dx} \rightarrow \text{shear flow} \)

between rebar and compression zone

when shear flow \( \frac{dT}{dx} = 0 \), but \( Jd \) is variable

\[
V = \frac{d (Jd)}{dx} \cdot T
\]
arch action

\[
C = \text{Compression} \quad Jd \text{ (varies)} \quad T = \text{Tension}
\]
Beam failure w/o shear reinforcement

(a) Beam.

(b) Moments at cracking and failure.

(c) Shear at cracking and failure.
Failure modes when a/d is 0.5 - 2.0

Types of failure:
1. Anchorage failure
2. Bearing failure
3. Flexural failure
4. Failure of compression strut

Loss of bond due to splitting crack.

(a) Shear-tension failure.

(b) Shear compression failure.

Failure modes when a/d is 1.5 - 2.5
Beams w. shear reinforcement

$V_a$: aggregate interlock
$V_d$: dowel action
$V_{cz}$: uncracked concrete
$V_s$: steel
\[ V_c = 2 \left(1 + \frac{Nu}{2000 + 1g}\right) \sqrt{F_{\text{bud}} b} \]

\[ V_s = \frac{Amf_{yd}}{s} \]

\[ V_n = V_c + V_s \]