Flexure and Shear

Flexure - behavior limit state

(a) Stage A — Before cracking.

(b) Stage B — After cracking, before yield, service load.

(c) Stage C — Failure.

Diagram showing flexural behavior with stages of cracking and failure.
(a) Beam.

(b) Bending moment diagram.

(c) Free body diagrams showing internal moment and shear force.

(d) Free body diagrams showing internal moment as a compression-tension force couple.
$C - T = 0 \quad \text{or} \quad C = T$

$M = T \cdot j d$

**MOMENT - CURVATURE**

- **D = Reinforcement yields**
- **C = Service load**
- **B = Cracking**
- **E = Failure**

**Axis of zero strain**

Curvature, $(1/r)$, (1/in.)
ultimate concrete strain

Compressive strength (MPa)

Ultimate flexural strain, $\varepsilon_{cu}$

Compressive strength (psi)

- Design value
- Columns
- Beams

(a) Ultimate strain from tests of reinforced members.

$\varepsilon_c = 0.003$

$C = k_1 k_3 f'_c b c$,

$M = k_1 k_3 (1 - k_2) f'_c b c^2$

$b$: beam width
ACI approach

\[ k_3 = \alpha \quad a = \beta c \quad (k_2 \cdot c = \frac{a}{2}) \]

\[ f' = 4,000 \text{ psi} \quad \beta_1 = 0.85 \]
\[ 4 \text{ to } 8,000 \text{ psi} \quad 1.05 - 0.05 \cdot \frac{f'}{1,000} \]
\[ 7,800 = 0.65 \]

\[ C = \alpha, \beta, f' \cdot b \cdot c \]

\[ M = \alpha, \beta, (1 - \frac{\beta_1}{2}) f' \cdot b \cdot c^2 \]
Example: Flexural capacity

\[
P_c = 3000 \text{ psi} \\
fy = 60 \text{ ksi}
\]

\[
A_s = (3)(0.79) = 2.3
\]

(a) Cross section.

(b) Elevation.

(c) Strain distribution.
\[ T = A_s f_y = 142,200 \text{ lb} \& \ C = T \]
\[ C = 0.85 f'_c \left( \frac{a^2}{2} \right) \quad a = \sqrt{\frac{142,200 \times 2}{0.85 \times 7000}} = 10.58'' \]

Check if steel yielded

\[ \frac{\varepsilon_s}{d - c} = \frac{0.003}{c} \]
\[ \varepsilon_s = 0.003 \left( \frac{22 - 12.42}{12.42} \right) \]
\[ = 0.00231 \]

\[ \varepsilon_y = \frac{60,000}{29,000,000} = 0.00207 \]

\[ \therefore \text{steel did yield} \]

\[ M_n = C \cdot Jd \quad Jd = d - \frac{2a}{3} \]
\[ M_n = A_s f_y \left( d - \frac{2a}{3} \right) \]
\[ = (2.37)(60,000)(22 - \frac{2 \times 10.58}{3}) \]
\[ = 2.127 \times 10^6 \text{ in-lb} \]

or \[ M_n = 177 \text{ ft-kips} \]
S/axial failure mode (strain based)

\[
\frac{c_{TCL}}{d_t} = \frac{0.003}{0.003 + 0.005}
\]

(a) Section

(c) Strains at compression-controlled limit

(d) Limiting strains for a beam

(e) Strains at tension-controlled failure

beam reinforcement limit

\( \varepsilon_s = 0.00231 \) is less than beam reinforcement limit.

318.99 did not have the \( \varepsilon_s \leq 0.004 \) limit (10.3.5).

318-02 \( \rho \leq \frac{3}{4} \rho_b \) \( \varepsilon_s \leq 0.00375 + 0.1\rho_b \) close to 0.004
(a) Arrangement of bars in two layers (ACI Section 7.6.2).

(b) Minimum bar spacing and cover limits in ACI Code.

(c) Maximum spacing of flexural reinforcement in slabs (7.6.5).